1. Suppose $X_1, ..., X_{10}$ is a random sample from the standard normal distribution. Find the value of c in each part with brief explanation.

(a) **(2 pts)** $P\left(\dfrac{X_1}{\sqrt{\sum_{i=2}^{10} X_i^2}} \leq c\right) = 0.95$

(b) **(2 pts)** $P\left(\dfrac{\sum_{i=1}^{4} X_i^2}{\sum_{i=5}^{10} X_i^2} \leq c\right) = 0.975$

Solution:

(a) Since $X_1, ..., X_{10}$ is a random sample from $N(0,1)$, we have $\sum_{i=2}^{10} X_i^2 \sim \mathcal{X}^2_{(9)}$ and is independent of X_1. Hence, by the definition of the t-distribution,

$$\frac{X_1}{\sqrt{\frac{\sum_{i=2}^{10} X_i^2}{9}}} = \frac{3X_1}{\sqrt{\sum_{i=2}^{10} X_i^2}} \sim t_{(9)}$$