1. Let $X_1, X_2, \ldots, X_n$ denote a random sample from the Pareto distribution with the following probability density function

$$f_X(x;\theta) = \begin{cases} \theta x^{-(\theta+1)} & \text{if } \ x > 1, \\ 0 & \text{otherwise.} \end{cases}$$

where $\theta > 0$ is an unknown parameter.

(a) **(3 pts)** Prove that the maximum likelihood estimator of θ is

$$\hat{\theta}_{\text{MLE}} = \frac{n}{\sum_{i=1}^{n} \log X_i}.$$

(b) **(3 pts)** Compute the Crámer-Rao Lower Bound (CR-LB) by showing that the Fisher information in a sample of size n is given by

$$I_n(\theta) = \frac{n}{}$$