1. Let $X_1, ..., X_{10}$ be a random sample from a standard normal distribution. Determine the probability distribution of W in each part. Justify your answers.

(a) **(2 pts)** $W = \dfrac{3X_1}{\sqrt{\sum_{i=2}^{10} X_i^2}}$.

(b) **(2 pts)** $W = \dfrac{\sum_{i=2}^{10} X_i^2}{9X_1^2}$.

Solution:

(a) Since $Z = X_1 \sim N(0,1)$, and by a theorem, $U = \sum_{i=2}^{10} X_i^2 \sim \chi^2_9$, and further Z and U are independent, by the definition of the t distribution,

$$W = \frac{3X_1}{\sqrt{\sum_{i=2}^{10} X_i^2}} = \frac{Z}{\sqrt{U/9}}$$