1. Suppose that $X_1, ..., X_n$ is a random sample from a $N(\mu, 1)$ distribution with the probability density function

$$f(x|\mu) = \frac{1}{\sqrt{2\pi}} e^{-(x-\mu)^2/2}, \quad \text{for} \quad x \in \mathbb{R}.$$

Further, the prior distribution of μ is a standard normal distribution.

(a) **(3 points)** Determine the posterior distribution of μ.

(b) **(2 points)** Use part (a) to find the posterior mean.