STA257 L0201, Midterm #2, Nov. 12, 2025

[SOLUTIONS]

1. **[5]** Suppose $X \sim \text{Binomial}(12, 1/3)$ and $Y = \min\{X, 10\}$.

Compute the probability function $p_Y(y)$ of Y, for <u>all</u> $y \in \mathbf{R}$.

Solution: *Notice that* $Y = 10$ *if* $X \geq 10$. *So, we calculate*

$$\begin{aligned} p_Y(10) &= \mathrm{P}(X \geq 10) = \sum_{k=10}^{12} \mathrm{P}(X = k) \\ &= \sum_{k=10}^{12} \binom{12}{k} \left(\frac{1}{3}\right)^k \left(1 - \frac{1}{3}\right)^{12-k} \\ &= \frac{12!}{(10!)(2!)} \left(\frac{1}{3}\right)^{10} \left(\frac{2}{3}\right)^2 + 12 \left(\frac{1}{3}\right)^{11} \left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)^{12} \\ &= \frac{289}{531,441}. \end{aligned}$$