STA257 L0201, Midterm #1, Oct. 6, 2025
[SOLUTIONS]

1. There are two boxes. The Left box contains 5 Red balls and 2 Green balls. The Right box contains 2 Red balls and 4 Green balls. One of the boxes is chosen uniformly (probability 1/2 each). Then, one of the balls is selected uniformly from the chosen box.

(a) [5] Compute P(Red), i.e. the probability that the selected ball is Red.

Solution: *Notice that choosing the Left or Right box, i.e. the events* {Left} *and* {Right}*, form a partition such that* $\mathrm{P}(\mathrm{Left}) = \mathrm{P}(\mathrm{Right}) = 1/2 > 0$*. So, by the law of total probability,*

$$\begin{aligned}
\mathrm{P}(\mathrm{Red}) &= \mathrm{P}(\mathrm{Left})\mathrm{P}(\mathrm{Red}|\mathrm{Left}) + \mathrm{P}(\mathrm{Right})\mathrm{P}(\mathrm{Red}|\mathrm{Right}) \\
&= \left(\frac{1}{2}\right)\left(\frac{5}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{6}\right) \\
&= \frac{11}{21} \approx 0.5238.
\end{aligned}$$

(b) [5] Compute $\mathrm{P}(\mathrm{Left}\,|\,\mathrm{Red})$, i.e. the conditional probability that the Left box was