**1.** Let $X$ be a random variable having a well-defined moment generating function $M_X(t)$ for $t$ in some neighbourhood of zero.

Prove that

$$P(X \geq a) \leq e^{-ta} M_X(t),$$

and

$$e^{tE[X]} \leq M_X(t),$$

where $t > 0$, and $a \in \mathbb{R}$ is a constant.

**Hint:** You may use the Markov and Jensen's inqualities to prove them.

Solution: For the first inequality, since $t > 0$, and the exponential function is an increasing and positive function, starting from the left-hand side, we have

$$P(X \geq a) = P(tX \geq ta) = P(e^{tX} \geq e^{ta}) \leq \frac{E(e^{tX})}{e^{ta}} = \frac{M_X(t)}{e^{ta}} = e^{-ta} M_X(t)$$

Note that the inequlaity follows from the Markov's inequality.