PHY250 Test 2
SOLUTIONS

1. **[8 marks]** Two small metal spheres (radius R) are far apart. Their centers are separated by a distance d. No other metal is nearby. Find the (approximate) capacitance between the two spheres.

   Hint: If the spheres are far enough away from each other ($d \gg R$) then it is safe to assume that they both have uniform surface charge densities (σ) when they are charged.

   Since the spheres are metal, they are conductors, and the charge will be on the surface. Since they are far from each other, the surface charge is uniform. By Gauss's Law, this is equivalent to a point charge at the center of each sphere. If we put the sphere with negative charge to be centered at the origin and the positively-charged sphere cenetered at $x = d$, and only worry about finding $\vec{E}(x)$ on the x-axis between the charges, we find

$$\vec{E}(x) = -\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{x^2} + \frac{1}{(d-x)^2}\right)\hat{x}$$

   The capacitance will be $C = Q/V$. We choose to find V by integrating $\vec{E}$ along the x-axis going from the negatively-charged surface to the positively-charged surface:

$$\int_{(-)}^{(+)} \vec{E}\cdot d\vec{\ell} = \int_R^{d-R} \left(\frac{1}{x^2} + \frac{1}{(d-x)^2}\right) dx$$