Math 1C03 Midterm - Version A

Section 02

October 24, 2022

1. (8 pts) Are the following statements true or false? If true, provide a proof, and if false, provide a counterexample and justify why your counterexample works.

   (i) $(\forall y \in \mathbb{Z})(\exists x \in \mathbb{Z})(y = x^2)$

   This statement says that for all y in $\mathbb{Z}$, there is x in $\mathbb{Z}$ such that x is a square root of y. This is **false**; for a counterexample, take any negative number, or any non-square positive integer, like the prime numbers. Since x^2 is positive for all integers $x \in \mathbb{Z}$, $y = -1$ is a counterexample to the statement.

   (ii) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(y = x^2)$

   This statement says that for all x in $\mathbb{Z}$, there is y in $\mathbb{Z}$ such that y is x^2. This is **true**