MATC44 - Practice Problem Set 4

These problems are not for credit and should not be handed in. While working through these problems, be sure to provide ample justification (e.g., a proof, logical argument or counter-example)!

Q1. A connected planar graph has degree sequence $2, 2, 3, 4, 4, 5$. How many faces does it have?

Q2. Does the following Corollary to Euler's formula for planar graphs also hold for disconnected planar graphs?

Corollary. Let G be a connected planar graph with $v \geq 3$ vertices and e edges. Then $e \leq 3v - 6$.

Hint: If G is disconnected and has $c \geq 1$ components, then a more general version of Euler's formula is $v - e + f = 1 + c$ (you may use this fact without proof).

Q3. For each $n \geq 3$, define the graph H_n to have vertex set $V(H_n) = \{3, 4, \ldots, n\}$ and suppose

$$uv \in E(G) \text{ if and only if either } v|u \text{ or } u|v.$$

Here, $x|y$ means x divides y. For example, H_{12} is shown below.