MATC44 - Fall 2025
Assignment 1 - Solutions

Due: September 18, 2025

Instructions: You may collaborate with your peers (and professor) on homework problems related to the course, however, you must write up your own solutions independently. Generative AI tools are not permitted for solving or drafting any part of an assignment. Full credit requires correct solutions with complete justification (e.g., a proof, logical argument or counter-example).

Q1 (6 marks)

Use the pigeonhole principle to prove that among any 25 points on the unit circle, at least 5 have all pairwise distances strictly less than 1.

(The distance between two points on a circle of radius r subtending a central angle θ is $2r \sin\left(\frac{\theta}{2}\right)$.)

Solution. Consider 25 points $P_1, \ldots, P_{25}$ on the unit circle. For $1 \leq i \leq 25$, label P_i by $(\cos\theta_i, \sin\theta_i)$ where $\theta_i \in [0, 2\pi)$. Partition $[0, 2\pi)$ into six half-open intervals of length $\pi/3$: