University of Toronto Scarborough Campus
Department of Computer and Mathematical Sciences

Problems

Q1. Find the critical points of the given function and then determine whether they are local maxima, local minima, or saddle points.

(a) $f(x, y) = x^2 - y^2 + xy$

(b) $f(x, y) = x^2 + y^2 + 2xy$

(c) $f(x, y) = e^{1+x^2-y^2}$

Q2. Let $f(x, y, z) = x^2 + y^2 + z^2 + kyz$.

(a) Verify that $(0, 0, 0)$ is a critical point for f .

(b) Find all values of k such that f has a local minimum at $(0, 0, 0)$.

Q3. Find the point on the plane $2x - y + 2z = 20$ nearest to the origin.

Q4. Show that the rectangular parallelepiped with fixed surface area and maximum volume is a cube.

Q5. Determine the nature of the critical points of $f(x, y) = x^3 + y^2 - 6xy + 6x + 3y$.

Q6. Find the absolute maximum and minimum values for $f(x, y) = \sin x + \cos y$ on the rectangle R defined by $0 \leq x \leq 2\pi$, $0 \leq y \leq 2\pi$.