University of Toronto Scarborough Campus
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Problems

Q1. (a) Let $f\colon \mathbb{R}^3 \to \mathbb{R}$, $\mathbf{x}_0 \in \mathbb{R}^3$. If $\mathbf{v}$ is a unit vector in $\mathbb{R}^3$, show that the maximum value of the directional derivative of f at $\mathbf{x}_0$ along $\mathbf{v}$ is $\|\nabla f(\mathbf{x}_0)\|$.

(b) Let $f(x, y, z) = x^3 - y^3 + z^3$. Find the maximum value for the directional derivative of f at the point $(1, 2, 3)$.

Q2. Compute the gradient ∇f for each of the following functions:

(a) $f(x, y, z) = 1/\sqrt{x^2 + y^2 + z^2}$

(b) $f(x, y, z) = xy + yz + xz$

(c) $f(x, y, z) = \dfrac{1}{x^2 + y^2 + z^2}$

Q3. A function $f\colon \mathbb{R}^n \to \mathbb{R}$ is called an *even* function if $f(\mathbf{x}) = f(-\mathbf{x})$ for every $\mathbf{x} \in \mathbb{R}^n$. If f is differentiable and even, find $\mathbf{D}f$ at the origin.

Q4. For each of the following, find the maximum and minimum values attained by the function f along the path $\mathbf{c}(t)$:

(a) $f(x, y) = xy$; $\mathbf{c}(t) = (\cos t, \sin t)$; $0 \le t \le 2\pi$