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Problems

Q1. Prove the following identities in $\mathbb{R}^n$.

(a) $\|\mathbf{x}+\mathbf{y}\|^2 - \|\mathbf{x}-\mathbf{y}\|^2 = 4(\mathbf{x}\cdot\mathbf{y})$

(b) $\|\mathbf{x}+\mathbf{y}\|^2 + \|\mathbf{x}-\mathbf{y}\|^2 = 2\|\mathbf{x}\|^2 + 2\|\mathbf{y}\|^2$

Q2. Consider the parallelpiped P spanned by the following vectors:

$$(0,1,0), (1,1,1), (0,2,0), (1,4,1).$$

(a) Use the determinant to find the volume of the parallelpiped P.

(b) Write P as a set $P = \{\mathbf{x} : \boxed{\text{fill in this blank}}\}$.
You may need to introduce additional variables a, b, c to write the set P.

Q3. Prove the following "reverse" triangle inequality in $\mathbb{R}^n$.

$$\|\mathbf{v}-\mathbf{w}\| \geq |\|\mathbf{v}\| - \|\mathbf{w}\||$$

Q4. Consider the set $U = \mathbb{R}^2 \setminus \{(0,0)\}$. Prove that U is open.

Q5. Consider the set $U = \{(x,y) : |y| < 1\}$. Prove that U is open.