As some of you requested, here is a sample of midterm test for MATB24.

1. (13 points)

a) [**6 points**] Give the definition of a field $\mathbb{F}$.

b) [**7 points**] Let V be a vector space over $\mathbb{Z}_3$ with a basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Suppose a new set of vectors is defined as follows: $\mathbf{w}_1 = \mathbf{v}_1 + 2\mathbf{v}_2$, $\mathbf{w}_2 = \mathbf{v}_2 + 2\mathbf{v}_3$, $\mathbf{w}_3 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$. Prove or disprove that $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ forms a basis of V.

2. (8 points) Let T: $M_2(\mathbb{R}) \to P_3(\mathbb{R})$ be a linear transformation from to defined by

$$T\left(\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\right) = x^3 + x,\ T\left(\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}\right) = x^2 + 1,\ T\left(\begin{bmatrix}1 & -1\\ 0 & 0\end{bmatrix}\right) = x^3 - 2,\ T\left(\begin{bmatrix}0 & 0\\ -1 & 1\end{bmatrix}\right) = x^2 + x.$$

Find $T\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)$.

3. **(12 points)** Let T: $M_2(\mathbb{R}) \to P_3(\mathbb{R})$ be a linear transformation defined by

$$T\left(\begin{bmatrix}a & b\\ c & d\end{bmatrix}\right) = ax^3 + bx^2 + cx + d\ .$$

Let $\beta = \left\{\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}, \begin{bmatrix}1 & -1\\ 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\ -1 & 1\end{bmatrix}\right\}$ and $\gamma = \{(x+1)^3,\ (x+1)^2,\ x+1,\ 1\}$ be ordered bases for $M_2(\mathbb{R})$ and $P_3(\mathbb{R})$, respectively.

a) Compute the matrix $[T]_\beta^\gamma$;