A67 Winter 2024: Midterm 2 (solution)

Question 1

Let (Ω, p) be a probability space representing a uniformly random choice, and let $E_1, ..., E_n$ be events. Prove by the contrapositive method that:

$$\left(\forall i \in [n],\ \ \exists j \in [n],\ \ |E_i \cap E_j| < |E_i|\right) \implies \left(\forall i \in [n],\ \ \Pr[E_i] \neq \Pr[E_1 \cap E_2 \cap ... \cap E_n]\right)$$

In your proof, write down the precise contrapositive statement that you are proving, explicitly and fully. *(25 pts)*

(You may use the fact that for any two sets A, B, if $A \subseteq B$ then $|A| \leq |B|$.)

We prove the contrapositive statement:

$$\left(\exists i \in [n],\ \ \Pr[E_i] = \Pr[E_1 \cap E_2 \cap ... \cap E_n]\right) \implies \left(\exists i \in [n],\ \ \forall j \in [n],\ \ |E_i \cap E_j| \geq |E_i|\right)$$