MATA22 Practice Term Test 1

1. Prove that, for all $z \in \mathbb{C}$,

$$|z| \leq |Re(z)| + |Im(z)| \leq \sqrt{2}|z|$$

Give examples to show that either inequality may be an equality.

2. Let $V = \{A | A \in M_{n\times n}(\mathbb{R})\}$. Let $P : V \times V \longrightarrow \mathbb{R}$ be defined as

$$P(A, B) = trace(A^T B).$$

(a) Show $P(A, A) \geq 0$, $P(A, B) = P(B, A)$ and $P(A + \alpha B, C) = P(A, C) + \alpha P(B, C)$ where $\alpha \in \mathbb{R}$. (ie: Check if P is a linear map.)

(b) Show that for every symmetric matrix D and skew symmetric matrix S,, we have

$$P(D, S) = 0$$