Solution to Assignment #11

1. Section 5.3: Please do parts (a), (c), (d) for questions 1, 2, 4.

    1. Let the sequence $a_0, a_1, a_2, \ldots$ be given by $a_0 = 0, a_1 = I$, and $a_k = \dfrac{(a_{k-1} + a_{k-2})}{2}$ for $k \geq 2$.
    a. Find the matrix A that can be used to generate this sequence as we used a matrix to generate the Fibonacci sequence in Section 5.I.

Solution:

Since $\begin{array}{rcl} a_{k+1} & = & \frac{1}{2}a_k + \frac{1}{2}a_{k-1} \\ a_k & = & a_k \end{array}$, if $A = \begin{bmatrix} 1/2 & 1/2 \\ 1 & 0 \end{bmatrix}$ then :

$$A \begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix} = \begin{bmatrix} 1/2a_k + 1/2a_{k-1} \\ a_k \end{bmatrix} = \begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix}$$

c. & d. For the eigenvalues of A:

$$|A - \lambda I| = \begin{vmatrix} 1/2 - \lambda & 1/2 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - \frac{1}{2}\lambda - \frac{1}{2} = (\lambda - 1)(\lambda + \frac{1}{2}).$$

The eigenvalues of A are: $\lambda = \frac{1}{2}$ and $\lambda = 1$