TEXTBOOK: Linear Algebra
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READ: Sections 1.5, 1.6.

PROBLEMS:

1. In each case find an invertible matrix U such that $UA = R$ is in reduced row echelon form, and express U as a product of elementary matrices.

   a) $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 0 \end{bmatrix}$. b) $A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 3 & 1 & 1 & 2 \\ 1 & -3 & 3 & 2 \end{bmatrix}$.

2. Prove that every elementary matrix is invertible.

3. Prove that a square matrix A is invertible if and only if $\mathbf{x} = \mathbf{0}$ is the only the solution to $A\mathbf{x} = \mathbf{0}$.

4. Let A be an $n \times n$ skew symmetric matrix and I be the $n \times n$ identity matrix.
   a) Show that $A - I$ and $A + I$ are invertible. (Hint: Use question 3.)
   b) Show that if $B = (I - A)(I + A)^{-1}$ then $B^T B = I$.

5. Section 1.5: # 6, 8, 12, 14, 20, 23, 24, 26, 34.