University of Toronto Scarborough Campus
Department of Computer and Mathematical Sciences

These solutions are not the only "perfect" or "10/10" solutions to be followed as a model. They are examples of ways to solve these problems. There are many many alternative ways to solve these problems. We hope that these example solutions help you check your work.

Problems

Q1. Suppose that $A \in \mathbf{M}_{n\times n}(\mathbb{R})$.

(a) Prove that $\det(kA) = k^n \det(A)$.

Solution: Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be the rows of A, and let us express the determinant of A as a function of its rows by $\det(A) = f(\mathbf{v}_1, \ldots, \mathbf{v}_n)$ so that $f : (\mathbb{R}^n)^n \to \mathbb{R}$ is multilinear. Then

$$\det(kA) = f(k\mathbf{v}_1, \ldots, k\mathbf{v}_n) = k^n f(\mathbf{v}_1, \ldots, \mathbf{v}_n) = k \det(A).$$

(b) Under what conditions does $\det(A) = \det(-A)$?

Solution: By (a) above, $\det(-A) = (-1)^n \det(A)$, so $\det(A) = \det(-A)$ when n is even.