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These solutions are not the only "perfect" or "10/10" solutions to be followed as a model. They are examples of ways to solve these problems. There are many many alternative ways to solve these problems. We hope that these example solutions help you check your work.

Problems

Recall, from the Week 4 Tutorial Activity: if $U, W \subseteq V$ are subspaces such that every vector in $\mathbf{v} \in U + W$ can be written uniquely $\mathbf{w} = \mathbf{u} + \mathbf{w}$ with $\mathbf{u} \in U$ and $\mathbf{w} \in W$ then we say that the sum $U + W$ is direct and write $U + W = U \oplus W$.

Q1. Suppose that V is finite dimensional and let $T : V \to V$ be a linear map such that $T^2 = T$.

(a) Prove that $V = \text{Image}(T) \oplus \ker(T)$.

Solution:

1. Firstly, let us show that $\text{Image}(T) + \ker(T) = V$. To do this, fix an arbitrary vector $\mathbf{v} \in V$ and express $\mathbf{v} = T(\mathbf{v}) + [\mathbf{v} - T(\mathbf{v})]$. Clearly, $T(\mathbf{v}) \in \text{Image}(T)$, so we only need to prove