University of Toronto Scarborough Campus
Department of Computer and Mathematical Sciences

These solutions are not the only "perfect" or "10/10" solutions to be followed as a model. They are examples of ways to solve these problems. There are many many alternative ways to solve these problems. We hope that these example solutions help you check your work.

Problems

Q1. On Homework #1 you proved that $(\mathbb{R}^+, \boxplus, \boxdot)$ was a vector space. Recall that,

$$\mathbb{R}^+ = \{x : x > 0\} \qquad x \boxplus y = xy \qquad c \boxdot x = x^c.$$

Prove that the function $T : \mathbb{R}^+ \to \mathbb{R}$ given by $T(x) = \ln(x)$ is a linear transformation.

Solution: Just note that for every $x, y, k \in \mathbb{R}$ we have that:

$$T(x \boxplus y) = \ln(xy) = \ln(x) + \ln(y) = T(x) + T(y)$$

and

$$T(k \boxdot x) = \ln(x^k) = k \ln(x) = kT(x).$$