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These solutions are not the only "perfect" or "10/10" solutions to be followed as a model. They are examples of ways to solve these problems. There are many many alternative ways to solve these problems. We hope that these example solutions help you check your work.

Problems

Q1. Consider a plane $\mathcal{P} = \{\mathbf{x} : \langle \mathbf{n}, \mathbf{x} \rangle = 0\}$ defined by a non-zero normal vector $\mathbf{n} \neq \mathbf{0}$.

(a) Give a precise definition of the linear transformation:

$$\sigma_{\mathcal{P}} = \text{"reflection across the plane } \mathcal{P}\text{"}$$

Solution: Let $\mathbf{v}$ and $\mathbf{w}$ be two independent vectors in $\mathcal{P}$ such that $\langle \mathbf{v}, \mathbf{w} \rangle = 0$, and let's assume that $\|\mathbf{v}\| = \|\mathbf{w}\| = 1$. Consider now the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(\mathbf{u}) := \langle \mathbf{u}, \mathbf{v} \rangle \mathbf{v} + \langle \mathbf{u}, \mathbf{w} \rangle \mathbf{w} - \langle \mathbf{u}, \mathbf{n} \rangle \mathbf{n}.$$

Note that: