University of Toronto Scarborough Campus
Department of Computer and Mathematical Sciences

These solutions are not the only "perfect" or "10/10" solutions to be followed as a model. They are examples of ways to solve these problems. There are many many alternative ways to solve these problems. We hope that these example solutions help you check your work.

Problems

Q1. Consider the set of polynomials with real coefficients $\mathbf{P}(\mathbf{R})$. Formally,

$$\mathbf{P}(\mathbf{R}) = \bigcup_{n=0}^{\infty} \mathbf{P}_n(\mathbf{R}).$$

Prove that $\mathbf{P}(\mathbf{R})$ is NOT finite dimensional.

Solution: Note that the monomials $\{1, x, x^2, \ldots, x^n, \ldots\}$ form an infinite linearly independent set. If $\mathbf{P}(\mathbf{R})$ where spanned by finitely many vectors, say $\{p_1(x), \ldots, p_n(x)\}$, and k where the maximum between the degrees of the $p_i(x)$'s, then $x^{k+1} \in \text{span}\{1, \ldots, x^k\}$, which is a contradiction.