MAT237 Multivariable Calculus with Proofs

Problem Set 3 solutions

Due Thursday November 13, 2025 by 23:59 ET

Remark:

Q3a You could also choose your domain to be unbounded. Reasonable choices include all of $\mathbb{R}^2$ or $[0,\infty)^2$. In this case, you have to apply Lemma 2.8.14 instead of the Extreme Value Theorem to prove the existence of the global maximum in (3b), and once you have optimized the revenue by finding critical points in (3c), you have to verify that the global maximum corresponds to non-negative prices and quantities.

Q3b
- If you chose an unbounded domain for R in (3a), then you must explicitly state this fact and prove the hint, and you must cite Lemma 2.8.14. instead of the extreme value theorem.
- If your domain is unbounded, e.g., $[0,\infty)^2$, you have to explicitly show that $R(p_1,p_2)\to-\infty$ as $\|(p_1,p_2)\|\to\infty$. One way to do so would be the following: we have the upper bound

$$\begin{aligned} R(p_1,p_2) &= 340p_1+480p_2-10p_1p_2-15p_1^2-20p_2^2 \\ &\le 340p_1+480p_2-10p_1p_2-15p_1^2-15p_2^2 \\ &\le -10(p_1^2+p_2^2)+340p_1+480p_2 && \text{(since } -ab\le\tfrac{1}{2}(a^2+b^2)\text{)} \\ &\le -10(p_1^2+p_2^2)+480(p_1+p_2) && \text{(since } p_1\ge 0\text{)} \end{aligned}$$

The last expression tends to $-\infty$ as $\|(p_1,p_2)\|\to\infty$, so $\lim\limits_{\|(p_1,p_2)\|\to\infty} R(p_1,p_2)=-\infty$ by a version of