MAT237 Multivariable Calculus with Proofs

Problem Set 1 solutions

Due Thursday September 25, 2025 by 23:59 ET

Remark:

(2b) The set $\{(x,y) \in \mathbb{R}^2 : x = 0 \text{ or } y = 0\}$ can also be written as the union

$$\{(x,y) \in \mathbb{R}^2 : x = 0\} \cup \{(x,y) \in \mathbb{R}^2 : y = 0\}$$

(5a) One can also solve this problem without noticing that $p \in S$ automatically implies that $p \in S \cap B_\varepsilon(p)$ for all $\varepsilon > 0$ by carefully choosing a point to the left of (x_0, x_0^2) that still lies within S. The main difficulty is that $\left(x_0 - \frac{\varepsilon}{2}, x_0^2\right)$ might not lie in S.

To this end, after x_0 and ε have been given, one can define $d := \min\left(\frac{\varepsilon}{2}, \frac{x_0}{2}\right)$ and consider the point $(x_0 - d, x_0^2)$. By construction,

1. $0 < x_0 - d < x_0$ and
2. $|x_0 - d| = x_0 - d < \varepsilon$

Then 1. implies that $0 < (x_0 - d)^2 < x_0^2 = y$ which in turn implies that $(x_0 - d, x_0^2) \in S$ and 2. implies that $(x_0 - d, x_0^2) \in B_\varepsilon((x_0, x_0^2))$.