1. (10 marks) Let C be the intersection of the cylinder $x^2 + y^2 = 1$ in $\mathbb{R}^3$ and the surface $z = f(x, y)$ where $f : \mathbb{R}^2 \to \mathbb{R}$ is a C^1 function. Evaluate

$$\int_C \frac{1}{\sqrt{1 + (-y\partial_x f(x,y) + x\partial_y f(x,y))^2}}\, ds$$

Solution:

A parameterisation of C is

$$x = \cos t, \quad y = \sin t. \quad z = f(\cos t, \sin t) \qquad (0 \le t \le 2\pi).$$

In vector terms, the parameterisation is

$$\mathbf{c}(t) = (\cos t, \sin t, f(\cos t, \sin t)) \qquad (0 \le t \le 2\pi))$$

. The velocity vector is:

$$\mathbf{c}'(t) = (-\sin t, \cos t, -\sin t\, \partial_x f(\cos t, \sin t) + \cos t\, \partial_y f(\cos t, \sin t)),$$

and hence (using the identity $\cos^2 t + \sin^2 t = 1$)

$$\|\mathbf{c}'(t)\|^2 = 1 + (-\sin t\, \partial_x f(\cos t, \sin t) + \cos t\, \partial_y f(\cos t, \sin t)^2$$