MAT236 – Fall 2025

Assignment 3 (Due November 7th, 9:00 PM on Crowdmark)

Problems

1. (10 points) Let T be a triangle in the $\mathbb{R}^2$ plane whose vertices are $A_i = (x_i, y_i)$ with $i = 1, 2, 3$. Let D be the region inside the triangle. Prove that

$$\iint_D x\, dA = \frac{x_1 + x_2 + x_3}{3} \quad \text{and} \quad \iint_D y\, dA = \frac{y_1 + y_2 + y_3}{3}.$$

Solution. Using a translation, it is clear that we may assume without any loss in generality that $x_3 = y_3 = 0$. The set of points in the triangle can now be parametrized as follows:

$$(x, y) = u\,(x_1, y_1) + v\,(x_2, y_2) \quad 0 \le a + b \le 1 \quad \text{and} \quad a, b \in [0, 1].$$

Since the area of the triangle is one, the Jacobian for this change of coordinates $(x, y) \to (u, v)$ is 2. Therefore,

$$\iint_D x\, dA = 2 \int_0^1 \int_0^{1-u} (ux_1 + vx_2)\, dv\, du = \frac{1}{3}(x_1 + x_2).$$

The claim for the y-moment follows analogously.