MAT236 – Fall 2025

Assignment 2 (Due October 17th, 9:00 PM on Crowdmark)

Problems

1. (6 points) Determine the minimum value of the integral

$$\int_C z\,dx + dy + (x + 2z)\,dz$$

among all curves C with initial point $(0, 1, 0)$ and the end point on the z-axis.

Solution. Note that

$$F = \nabla V$$

where $V : \mathbb{R}^3 \to \mathbb{R}$ is defined by

$$V(x, y, z) = xz + y + z^2.$$

Therefore, given any curve C with starting point $p = (0, 1, 0)$ and end point $(0, 0, t)$, we have

$$\int_C z\,dx + dy + (x + 2z)\,dz = V(q) - V(p) = t^2 - 1,$$