MAT236 – Fall 2025

Assignment 1 (Due October 3rd, 9:00 PM on Crowdmark)

Problems

1. (6 points) Consider the surface in $\mathbb{R}^3$ given by the set of points (x, y, z) that satisfy

$$x^2 + y^2 + z^2 - xyz = 2.$$

   (a) Is it possible to write z as a differentiable function of x, y near $(x, y, z) = (1, 1, 1)$? If so, what would be the values of $\frac{\partial}{\partial x} z$ and $\frac{\partial}{\partial y} z$ at the point $(1, 1)$?

   (b) Repeat the previous part near the point $(\sqrt{2}, 0, 0)$.

Solution. (a) Let $F(x, y, z) = x^2 + y^2 + z^2 - xyz - 2$. Note that the point $(1, 1, 1)$ lies on the zero level set of the function $F : \mathbb{R}^3 \to \mathbb{R}$. Note also that $F \in C^1(\mathbb{R}^3)$. Finally, let us observe that

$$\frac{\partial}{\partial z} F = 2z - xy,$$

and therefore $\frac{\partial}{\partial z} F\big|_{(1,1,1)} = 1 \neq 0$. Thus, the implicit function theorem applies to deduce that there exists a neighborhood U of the point $(1, 1)$ in the plane and a function $g \in C^1(U)$ such that