1. Let S be a region in $\mathbb{R}^3$ given by

$$S = \{(x, y, z) \in \mathbb{R}^3 : \frac{(x-3)^2}{4} + \frac{(y-1)^2}{4} + \frac{(z+5)^2}{9} \geq 49\}$$

(a) Describe the following set in terms of its shape (i.e., how it looks like in $\mathbb{R}^3$) and the boundary of its shape.

(b) Recall that an object has rotational symmetry about an axis if no matter how this object is spun around this axis, it looks completely the same. Find a sextuple $(a_1, a_2, a_3, b_1, b_2, b_3) \in \mathbb{R}^6$ such that the series of transformations

1. shifting towards the positive x direction a_1 units
2. shifting towards the positive y direction a_2 units
3. shifting towards the positive z direction a_3 units
4. compressing in the x direction for b_1 times
5. compressing in the y direction for b_2 times
6. compressing in the z direction for b_3 times

allows the boundary you found to have rotational symmetry about the x axis but not the z axis, and the volume enclosed by the boundary stays the same.

(a):

The set is $\mathbb{R}^3$ with an ellipsoid scoped out. The ellipsoid is centered at $(3, 1, -5)$, and with semi-axis length 14 in the x and y directions, and 21 in the z direction. The boundary of the ellipsoid is contained in the set.