Problem 1. We know that there exists a basis β for $\mathbb{C}^2$ such that

$[T]_\beta^\beta$ = an upper triangular form. $= \begin{bmatrix} \lambda_1 & 3 \\ 0 & \lambda_2 \end{bmatrix}$

$T - \lambda I = \begin{bmatrix} 1-\lambda & 1 \\ -1 & 3-\lambda \end{bmatrix} = (1-\lambda)(3-\lambda)+1 \overset{let}{=} 0$

$\lambda^2 - 4\lambda + 4 = 0 \Rightarrow (\lambda-2)^2 = 0$ $\boxed{\lambda = 2}$

$T - 2I = \left[\begin{array}{cc|c} -1 & 1 & 0 \\ -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $-x_1 + x_2 = 0$. Let $x_2 = t$. Then $x_1 = t$.

$E_{\lambda=2} = \text{span}(\{(1,1)\})$

Let $\beta = \{(1,1), x\}$ where $x \in \mathbb{C}^2 - E_{\lambda=2}$.

Then $T(x) = 3(1,1) - 2x$.

$(T-2I)(x) = (3,3)$

Solve $\left[\begin{array}{cc|c} -1 & 1 & 3 \\ -1 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{cc|c} -1 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$ $-x_1 + x_2 = 3$.