problem 4 Sec 5.1 # 5 (a)

(a) Let $p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0$ where $a_i \in \mathbb{R}$ $0 \le i \le n$ $a_n \neq 0$

Suppose $p(z) = 0$. Then $a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 = 0$

$$\Rightarrow \quad \overline{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0} = \overline{0} = 0$$

$$\Rightarrow \quad \overline{a_n z^n} + \overline{a_{n-1} z^{n-1}} + \cdots + \overline{a_0} = 0$$

$$\Rightarrow \quad a_n (\bar{z})^n + a_{n-1} (\bar{z})^{n-1} + \cdots + a_0 = 0 \quad \text{because} \quad \overline{a_i} = a_i \text{ and } \overline{z^i} = (\bar{z})^i \quad 0 \le i \le n$$

$$\Rightarrow \quad p(\bar{z}) = 0$$

problem 3 $A = [a_1 \; a_2]$ is orthogonal $\Leftrightarrow$ $\{a_1, a_2\}$ is orthonormal

pf $A = [a_1 \; a_2]$ is orthogonal $\Leftrightarrow$ $A^t A = I$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \underbrace{[a_1 \; a_2]}_{\| \; \| \; \| \; \|} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \langle a_1, a_1 \rangle & \langle a_1, a_2 \rangle \\ \langle a_2, a_1 \rangle & \langle a_2, a_2 \rangle \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$