Problem 1

Sec 2.6 # 1(a) $T(p(x)) = xp'(x)$. Since $T(1) = x(1)' = x \cdot 0 = 0$ and $T(2) = x(2)' = x \cdot 0 = 0$,
$\ker(T) \neq \{0\}$. $\Rightarrow$ T is not injective $\Rightarrow$ not an isomorphism.

1(c) $T: P_2(\mathbb{R}) \to \mathbb{R}^3$ defined by $T(p(x)) = (p(0), p(1), p(2))$

Claim: T is injective.

pf. $\forall\ p(x) = a + bx + cx^2 \in \ker(T)$.

Then $(p(0), p(1), p(2)) = (0, 0, 0)$

$$\text{So} \begin{cases} a = 0 \\ a+b+c = 0 \\ a+2b+4c = 0 \end{cases} \Rightarrow a = 0,\ b = 0,\ c = 0 \Rightarrow p(x) = 0 \text{ polynomial}$$

$\Rightarrow \ker(T) = \{0\} \Rightarrow$ T is injective