Problem 1 Sec 1.1 #7 (b) and (c)

$F(\mathbb{R}) = \{f \mid f : \mathbb{R} \to \mathbb{R}\}$

(b) $f +' g = f - g$, $c \cdot f(x) = f(cx)$

$F \to$ axiom 1 : $(f +' g) +' h \neq f +' (g +' h)$. For example, $f=1, g=2, h=3$. $(f +' g) +' h = -4$ and $f +' (g +' h) = 2$

$F \to$ axiom 2 $f +' g \neq g +' f$ For example, $f=1, g=2$. Then $f +' g = 1-2 = -1$ $g +' f = 2-1 = 1$

$F \to$ axiom 3 fails because $f +' 0 = f$ but $0 +' f = -f$

$F \to$ axiom 4. fails because axiom 3 fails (no additive inverse)

axiom 5 holds: $c \cdot (f +' g)(x) = c \cdot (f-g)(x) = (f-g)(cx) = f(cx) - g(cx)$

$= c \cdot f(x) - c \cdot g(x) = (cf) +' (cg)$

$F \to$ axiom 6 $(c+d) \cdot f \neq c \cdot f + d \cdot f$. For example. $f(x) = e^x$. $c=1, d=2$

$(c+d) \cdot f(x) = 3 \cdot f(x) = f(3x) = e^{3x}$

$(c \cdot f + d \cdot f)(x) = 1 \cdot f(x) + 2 \cdot f(x) = f(x) + f(2x) = e^x + e^{2x}$