HW 9

Problem 1 Let $T : \mathbf{C}^2 \longrightarrow \mathbf{C}^2$ be defined by the matrix $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$.

Find a basis β such that $[T]_\beta^\beta = \begin{bmatrix} \lambda_1 & 3 \\ 0 & \lambda_2 \end{bmatrix}$ where λ_1 and λ_2 are eigenvalues of T.

Problem 2 Let $C(\mathbf{x}_1) = \{N^{l_1}(\mathbf{x}_1) \cdots, \mathbf{x}_1\}$ and $C(\mathbf{x}_2) = \{N^{l_2}(\mathbf{x}_2) \cdots, \mathbf{x}_2\}$ be cycles where N is a nilpotent mapping from a complex vector space V to V. Suppose $N^{l_1}(\mathbf{x}_1)$ and $N^{l_2}(\mathbf{x}_2)$ are linearly independent. Show that

(a) $C(\mathbf{x}_1) \cap C(\mathbf{x}_2) = \emptyset$
(b) $C(\mathbf{x}_1) \cup C(\mathbf{x}_2)$ is linearly independent. (Prove (b) when $l_1 = 2$ and $l_2 = 1$.)

Problem 3 sec6.2 #1(a) and #7(a)