HW 3

Problem 1 Let V be a n-dimensional vector space over $\mathbf{R}$ with a basis α. Show that the mapping from V to $\mathbf{R}^n$ defined by $[\mathbf{x}]_\alpha$ where $\mathbf{x} \in V$, is a linear mapping.

Notation: $[\mathbf{x}]_\alpha$ is the coordinates of $\mathbf{x}$ with respect to the basis α.

Problem 2 The linear transformation $T : P_2(\mathbf{R}) \longrightarrow P_1(\mathbf{R})$ is defined by $T(1) = 2 - 4x, T(x) = 1 - 2x$, and $T(x^2) = 3 - 6x$.

(a) Find the matrix with respect to the standard bases $\{1, x, x^2\}$ and $\{1, x\}$ of $P_2(\mathbf{R})$ and $P_1(\mathbf{R})$ respectively.

(b) Find $\mathrm{Ker}(T)$ and $\mathrm{Im}(T)$.

(c) Find a basis of $P_2(\mathbf{R})$ using $\mathrm{Ker}(T)$.

(d) Find a basis of $P_1(\mathbf{R})$ using $\mathrm{Im}(T)$.

Problem 3 sec2.1 #15 and find the matrix of $T|_U$ with respect to the basis $\alpha = \{\mathbf{u}_1, \mathbf{u}_2\}$ for U and the standard basis β for $\mathbf{R}$.