Q1 (a) T is not injective because $\ker(T) \neq \{0\}$

(b) $\dim(\ker(T)) + \dim(\mathrm{Im}(T)) = \dim(\mathbb{R}^4) = 4$

since $\dim(\ker(T)) = 1$, $\dim(\mathrm{Im}(T)) = 3$.

(c) If T is surjective, $\dim(\mathrm{Im}(T)) = \dim(W)$. Therefore, $\dim(W) = 3$.

Q2

$$T(0_V) = T(0_V + 0_V) = T(0_V) + T(0_V)$$
$$\Rightarrow T(0_V) + (-T(0_V)) = T(0_V) + T(0_V) + (-T(0_V))$$
$$\Rightarrow 0_W = T(0_V) + 0_W \Rightarrow 0_W = T(0_V)$$

Q3 (a) i) $\forall p(x), q(x) \in P_2(\mathbb{R})$, $T(p+q) = \begin{bmatrix} (p+q)(0) & (p+q)'(0) \\ (p+q)'(0) & (p+q)''(0) \end{bmatrix} = \begin{bmatrix} p(0)+q(0) & p'(0)+q'(0) \\ p'(0)+q'(0) & p''(0)+q''(0) \end{bmatrix}$

$$= \begin{bmatrix} p(0) & p'(0) \\ p'(0) & p''(0) \end{bmatrix} + \begin{bmatrix} q(0) & q'(0) \\ q'(0) & q''(0) \end{bmatrix} = T(p) + T(q)$$