MAT223 Midterm

Instructions

Please see the answer booklet for instructions and only write your solutions on the answer booklet.

A: Basic Concepts and Definitions

1. Determine the following and justify your answers:

   (a) (5 pts) To define a vector subspace of $\mathbb{C}^n$, we can simply replace $\mathbb{R}$ everywhere with $\mathbb{C}$ in the definition of a vector subspace of $\mathbb{R}^n$. Given this, is $\{(a,b,c,d,e) \in \mathbb{C}^5 : a^5 = b^5 = c^5\}$ a vector subspace of $\mathbb{C}^5$? (You can use the following two facts without giving any justification: $(\cos(\frac{\pi}{5}) + i sin(\frac{\pi}{5}))^5 = -1$ and $(1 + \cos(\frac{\pi}{5}) + i sin(\frac{\pi}{5}))^5 \neq 0$.)

   Sketch solution: No. Consider $u_1 = (1,1,1,0,0)$ and $u_2 = (-1,-1,\cos(\frac{\pi}{5}) + i\sin(\frac{\pi}{5}),0,0)$. These two are both in the set. But we know that $(-1+1)^5 = 0$ but $(1+\cos(\frac{\pi}{5})+i\sin(\frac{\pi}{5}))^5 \neq 0$, which implies $u_1 + u_2$ is not in the set. Thus, this set is not closed under addition.

   (b) (5 pts) Is the plane $P = \{(x,y,z) \in \mathbb{R}^3 : x + 2y + 3z = 0\} = span((0,3,-2),(2,-1,0))$ a vector subspace of $\mathbb{R}^3$?