[6] 1. (a) How many arrangements of the word REFERENCE contain the string FENCE?

Solution: There are $\dfrac{5!}{2!\ 2!}$ arrangements, obtained by treating FENCE as a single block.

(b) How many arrangements of the word REFERENCE contain FENCE but neither REF nor ERE?

Solution: Let A be the arrangements that contain both FENCE and REF, and let B be the arrangements that contain both FENCE and ERE. Since A and B may not be disjoint, we use PIE and compute the desired quantity, which is

$$\frac{5!}{2!\ 2!} - |A| - |B| + |A \cap B|.$$

Arrangements in A must have the block REFENCE, and two other letters R and E. Hence $|A| = 3!$.

Arrangements in B come in two cases: either it contains the block FENCERE, with two other letters R and E, or it contains the block FENCE, the block ERE, and one other letter R. In either case there are 3! arrangements. However, these cases overlap! The arrangement

$$\underline{\text{FENCERE}}\overline{\text{RE}}$$

satisfies both of these cases, and so by PIE we have $|B| = 3! + 3! - 1$.

Finally, $|A \cap B| = 3$ since the three arrangements