1. From a standard deck of cards, how many **three**-card hands are there that contain

[3] (a) exactly two different ranks in the hand?
(For example, the hand $2\diamondsuit$ $2\spadesuit$ $K\clubsuit$ is a valid hand as it contains exactly two ranks, 2 and K.)

Solution:
Since the hand has three cards and needs to have exactly two ranks, right away we know that one of the ranks must occur twice in the hand, and the other rank occurs once.
There are 13 options for the repeated rank, and $\binom{4}{2}$ ways to decide their suits; then, 12 options for the other rank, and 4 choices for its suit.
Thus there are $13 \cdot 12 \cdot \binom{4}{2} \cdot 4$ such three-card hands.

[4] (b) exactly two different ranks **or** exactly two different suits in the hand (or both)?

Solution: If A is the set of three-card hands that contain exactly two ranks and B is the set of three-card hands that contain exactly two suits, then the desired quantity is $|A \cup B|$.
By PIE, this is $|A \cup B| = |A| + |B| - |A \cap B|$.
We already have $|A|$ from part (a), while the cardinality $|B|$ is obtained very similarly.
For the three-card hand to have exactly two suits represented, one of the suits occurs twice in the hand. There are 4 options to pick the repeated suit and $\binom{13}{2}$ ways to decide the ranks for that suit. Then, there are 3 options