MAT137 Problem Set 6

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1. (*The Laplace Transform*). Let $f(x) : [0, \infty) \to \mathbb{R}$ be an integrable function. The **Laplace transform** of $f(x)$, sometimes denoted by $\widehat{f}(s)$, is defined by

$$\widehat{f}(s) := \int_0^\infty f(t)e^{-st}\,dt \tag{1}$$

Named after Pierre-Simon Laplace, the Laplace transform was one of the first of the so-called **integral transforms**. The properties of the Laplace transform make it invaluable for solving differential equations.

(a) Suppose there exists $a, b \geq 0$ such that for all $x \in [0, \infty)$, $|f(x)| \leq be^{ax}$. Show that