MAT137 Problem Set 4

Osama Farooqui & George Huang

1. In this problem, we will show that any continuous function $f : [a, b] \to \mathbb{R}$ is integrable.

   (a) Let $f : [a, b] \to \mathbb{R}$ be a bounded function. Prove that $f(x)$ is integrable iff for any $\varepsilon > 0$, there exists a partition P of $[a, b]$ such that

$$\mathcal{U}_P(f) - \mathcal{L}_P(f) < \varepsilon \tag{1}$$