MAT 137Y: Calculus with proofs
Assignment 1
Sample Solutions

1. Consider the following statement:
"Every (strictly) increasing function f with domain $\mathbb{R}$ satisfies: if x_1 and x_2 are real numbers such that $f(x_1) > 0$ and f is one-to-one on the interval $(x_1 - 1, x_1 + 1)$ and $0 < x_2 - x_1 < 1/2$, then
$[f(x_1 + 1/2)]^2 > f(x_2) > 0$. "

   (a) Negate this statement without using "no", "not", "false", etc.

   There is at least one (strictly) increasing function f with domain R which satisfies the property $\exists x_1, x_2 \in \mathbb{R}$ such that $f(x_1) > 0$ and f is one-to-one on the interval $(x_1 - 1, x_1 + 1)$ and $0 < x_2 - x_1 < 1/2$ AND $[f(x_2) \leq 0$ or $f(x_2) \geq [f(x_1 + 1/2)]^2]$.

   (b) Is this statement true or false? If it is true, prove it. If it is false, provide a counterexample