MAT 137Y: Calculus with proofs
Assignment 2
Sample Solutions

1. (*Note:* Before you attempt this problem, solve Problem 1 and 3 from Practice problems for Unit 2 on Quercus or Problem 3 and Problem 4 from the practice problems of test 1 on Quercus. Otherwise you may find this question difficult.)

   Sketch the graph of a function f that satisfies all 9 conditions below simultaneously. For this question, you do not need to prove or explain your answer, as long as the graph is correct and very clear.

   (a) The domain of f is $\mathbb{R}$.

   (b) $\lim\limits_{x\to 1} f(x) = 2$ and $\lim\limits_{x\to 1} f(f(x))$ does not exist, is not ∞, and is not $-\infty$.

   (c) $\lim\limits_{x\to 3} f(x) = -1$ and $\lim\limits_{x\to 3} f(f(x)) = -\infty$.

   (d) $\lim\limits_{x\to -3} f(x) = -1$ and $\lim\limits_{x\to -3} f(f(x)) = \infty$.

   (f) $\lim\limits_{x\to -1} f(f(x)) = 4$

   (g) $\lim\limits_{x\to 0} f(x) = 1$ and $\lim\limits_{x\to 0} \lfloor f(x) \rfloor$ does not exist

   (h) $\lim\limits_{x\to 5} f(x)$ does not exist and $\lim\limits_{x\to 5} [f(x)]^2 = 1$