Solutions to E1, E2 and F1 on the Practice Final Exam.

**E1.** **[2 marks]** Find the area in the finite region between the curves $f(x) = \sqrt{x}$ and $g(x) = x^2$. Include a calculation for the only two points of intersection between the graphs of the two functions. Then, set up a definite integral to find the area.

**Solution:** Equating $f(x) = g(x)$ and squaring both sides to get $x = x^4$, we see that the solutions of $x^4 - x = x(x^3 - 1)$ that are non-negative and real are $x = 0$ and $x = 1$ (note that $f(x)$ is only defined for $x \geq 0$ in the $xy$-plane). Thus, $f(x)$ and $g(x)$ intersect at $(0, 0)$ and $(1, 1)$.
By either sketching the functions or computing sample values of $f(x) - g(x)$, we see that $f(x) \geq g(x)$ on $x \in [0, 1]$. Thus, the definite integral computing the area in the finite region between $f(x)$ and $g(x)$ is

$$\begin{aligned}\int_0^1 (f(x) - g(x))dx &= \int_0^1 (\sqrt{x} - x^2)dx \\ &= \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)_0^1 \\ &= \frac{2}{3} - \frac{1}{3} \\ &= \frac{1}{3}.\end{aligned}$$