MAT133
WRITTEN HOMEWORK 2 **Due: Friday, June 9, 2023 at 11:59pm**

All solutions to be written in appropriate details. Please submit your solution to Crowdmark.

NOTE: *This version contain solutions to the problems. Solutions are not written in best details, so students are expected to fill in some of these gaps in their submissions.*

Question 1. Assume that you have the usual 52 card deck, 13 ranks and 4 suits.

a. How many six-card hands have at least one A (ace) **or** at least one ♡ (heart) in them?

Solution Let S be the set of all possible six-card hands, and A be the set of six-card hands with at least one ace or at least one heart. Then, the complement of A, $A^c = S \backslash A$, is the set of six-card hands with no aces **and** no hearts. Then, the number of elements in S ignoring the order is

$$_{52}C_6 = |S| = |A| + |A^c|.$$

Hence, to find $|A|$ we need to find $|A^c|$. By the definition, A^c is the set of six-card hands with cards taken from the deck with no aces and no hearts included. There are 4 aces and 13 hearts, but $A\heartsuit$ is in both, so the number of cards in the deck to choose for A^c is $52 - 4 - 13 + 1 = 36$. So the number of elements in A^c is equal to the number of ways to choose 6 cards out of 36 ignoring the order, $_{36}C_6$. Then,

$$|A| = |S| - |A^c| =_{52} C_6 -_{36} C_6.$$

b. How many six-card hands have at least one A **and** at least one ♡ in them?