ECO220Y1Y, Test #2, Prof. Murdock SOLUTIONS

(1) (a) $P(S3\ \&\ W) = 0.0521$ [Note the order of events S3 and W does not matter for a joint probability.]

(b) $P(S2) = 0.3446$

(c) $P(S4 \mid T) = \frac{P(S4\ \&\ T)}{P(T)} = \frac{0.0976}{0.5716} = 0.171$

(d) $P(W \mid S1) + P(Y \mid S1) = \frac{P(W\ \&\ S1)}{P(S1)} + \frac{P(Y\ \&\ S1)}{P(S1)} = \frac{0.0645}{0.3056} + \frac{0.0637}{0.3056} = 0.420$ [Also, $1 - P(T \mid S1) = 0.420$]

(e) Events S4 and Y are independent if and only if P(S4 | Y) = P(S4) [or you could equivalently check that P(Y | S4) = P(Y)]. P(S4 | Y) = 0.0209/0.2228 = 0.094 and P(S4) = 0.1413. These are unequal so Events S4 and Y are *not* independent.

(2) It is a linear combination of the original variables: $X_A = \frac{X_H + X_C}{2}$. The Laws of Expectation for linear combinations work for sample statistics so $\bar{X}_A = \frac{\bar{X}_H + \bar{X}_C}{2} = \frac{6.395189 + 6.790378}{2} = 6.59.$

$$V[X_A] = V\left[\frac{X_H + X_C}{2}\right] = \frac{1}{4}V[X_H] + \frac{1}{4}V[X_C] + 2 * \frac{1}{2} * \frac{1}{2} * SD[X_H] * SD[X_C] * CORR[X_H, X_C]$$
$$= 0.25(1.835994)^2 + 0.25(2.159353)^2 + 0.5 * 1.835994 * 2.159353 * 0.2071 = 2.419$$

$$SD[X_A] = \sqrt{2.419} = 1.555$$