Question 1. [4 MARKS] GCD

Recall the following facts about gcd and divisibility:

$$\forall m, n \in \mathbb{N}^+, \gcd(m,n) \in \mathbb{N}^+ \wedge \gcd(m,n) \mid m \wedge \gcd(m,n) \mid n \wedge \forall d \in \mathbb{N}^+, d \mid m \wedge d \mid n \Rightarrow d \leq \gcd(m,n) \quad \text{(GCD1)}$$

$$\forall m, n, g \in \mathbb{N}^+, \Big(g \mid m \wedge g \mid n \wedge \forall d \in \mathbb{N}^+, d \mid m \wedge d \mid n \Rightarrow d \leq g\Big) \Rightarrow g = \gcd(m,n) \quad \text{(GCD2)}$$

$$\forall m, n, d, s, t \in \mathbb{Z}, d \mid m \wedge d \mid n \Rightarrow d \mid sm + tn \quad \text{(C)}$$

$$\forall d, n \in \mathbb{N}^+, d \mid n \Rightarrow d \leq n \quad \text{(B)}$$

Prove that $\forall k \in \mathbb{N}, \gcd(9k+11, 4k+5) = 1$.

Use only the facts about gcd *and divisibility provided in this question—if you want to use any other property of* gcd *or divisibility in your answer, you must* ***prove*** *it first.*

MINIMAL SOLUTION

Let $k \in \mathbb{N}$.

By (C), $\gcd(9k+11, 4k+5) \mid (-4) \cdot (9k+11) + 9 \cdot (4k+5) = -36k - 44 + 36k + 45 = 1$
($\because \gcd(9k+11, 4k+5) \mid 9k+11$ and $\gcd(9k+11, 4k+5) \mid 4k+5$, by (GCD1)).

Then by (B), $\gcd(9k+11, 4k+5) \leq 1$, so $\gcd(9k+11, 4k+5) = 1$.