Recall the following properties given in WS08Wed and WS08Mon, for all functions $f, g, h : \mathbb{N} \to \mathbb{R}^{\geq 0}$ and all constants $a_0 < b_0 \in \mathbb{R}^+$, $a_1 < b_1 \in \mathbb{R}^{>1}$. We also include two more properties related to floor and ceiling. (NOTE. Technically, the functions f, g, h only need to be *eventually* defined and non-negative, as pointed out on WS08Wed.)

$$a_0 \in \Theta(b_0) \qquad \text{(CONST)}$$

$$\log_{a_1} \in \omega(1) \qquad \text{(CONST-LOG)}$$

$$\log_{a_1} \in \Theta(\log_{b_1}) \qquad \text{(LOG)}$$

$$n^{a_0} \in \omega(\log) \qquad \text{(LOG-POLY)}$$

$$n^{b_0} \in \omega(n^{a_0}) \qquad \text{(POLY)}$$

$$a_1^n \in \omega(n^{a_0}) \qquad \text{(POLY-EXP)}$$

$$b_1^n \in \omega(a_1^n) \qquad \text{(EXP)}$$

$$f \in \omega(g) \Rightarrow g \in \mathcal{O}(f) \qquad (\omega\text{-}\mathcal{O})$$

$$g \text{ is eventually positive} \wedge f \in \omega(g) \Rightarrow f \notin \mathcal{O}(g) \qquad (\omega\text{-NOT}\mathcal{O})$$

$$f \in \Omega(g) \Leftrightarrow g \in \mathcal{O}(f) \qquad \text{(DEF-}\Omega\text{)}$$

$$f \in \Theta(g) \Leftrightarrow f \in \mathcal{O}(g) \wedge f \in \Omega(g) \qquad \text{(DEF-}\Theta\text{)}$$

$$a_0 \cdot f \in \Theta(f) \qquad \text{(CONST-FACTOR)}$$

$$f \in \mathcal{O}(g) \wedge g \in \mathcal{O}(h) \Rightarrow f \in \mathcal{O}(h) \qquad \text{(TRANSITIVITY-}\mathcal{O}\text{)}$$